

The Complex Number System

DEFINITION of i

$$\sqrt{-N} = \sqrt{N}i$$

*If there is a negative inside the radical take it OUT of the radical as "i"

$$\sqrt{-8} = \sqrt{8}i = \sqrt{4 * 2}i = 2\sqrt{2}i$$

$$i^2 = -1$$

*If you ever get an i^2 in your problem, always replace it with a (-1)

$$3i^2 = 3(-1) = -3$$

STANDARD FORM

$$a + bi$$

Real number part

Imaginary number part

****Always break apart into the two pieces**

$$\frac{4 - 6i}{4} = \frac{4}{4} - \frac{6i}{4} = 1 - \frac{3}{2}i$$

ADD

$$(a + bi) + (c + di) = a + bi + c + di$$

$$= (a + c) + (b + d)i$$

*Add real number pieces and imaginary number pieces

$$(3 + 2i) + (5 - 6i) = 3 + 2i + 5 - 6i$$

$$= (3 + 5) + (2 - 6)i$$

$$= 8 - 4i$$

SUBTRACT

$$(a + bi) - (c + di)$$

$$= a + bi - c - di$$

$$= (a - c) + (b - d)i$$

*Distribute the negative to the second term and then combine like pieces

$$(3 + 5i) - (7 - 8i)$$

$$= 3 + 5i - 7 + 8i$$

$$= (3 - 7) + (5 + 8)i$$

$$= -4 + 13i$$

MULTIPLY

* Replace all i^2 with a (-1)

*Always simplify to Standard Form

$$(3 - 4i)(2 + 5i)$$

$$= 3(2) + 3(5i) + 2(-4i) + (-4i)(5i)$$

$$= 6 + 15i - 8i - 20i^2$$

$$= 6 + 7i - 20(-1)$$

$$= 6 + 7i + 20$$

$$= 26 + 7i$$

COMPLEX CONJUGATES

$(a + bi)$ complex conjugate is $(a - bi)$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$(3 - 2i)(3 + 2i) = 3^2 + 2^2 = 9 + 4 = 13$$

DIVIDE

1. Find complex conjugate of denominator
2. Multiply top & bottom by the complex conjugate
3. Simplify to Standard Form

$$\frac{2 + 4i}{5 - 2i} \cdot \frac{(5 + 2i)}{(5 + 2i)}$$

$$= \frac{10 + 4i + 20i + 8i^2}{5^2 + 2^2}$$

$$= \frac{10 + 24i - 8}{29}$$

$$= \frac{2}{29} + \frac{24}{29}i$$

SIMPLIFY POWERS OF i

$$i^n = i \text{ for}$$

$$n = 1, 5, 9, \dots$$

$$i^n = -1 \text{ for}$$

$$n = 2, 6, 10, \dots$$

$$i^n = 1 \text{ for}$$

$$n = 4, 8, 12, \dots$$

$$i^n = -i \text{ for}$$

$$n = 3, 7, 11, \dots$$

*Divide exponent by 4 and see how many times 4 goes into it evenly and find the remainder. The remainder value becomes new exponent to find the value of i

$$i^{39} = ?$$

$$\frac{39}{4} = 9 \text{ remainder of } 3 \text{ so use } i^3 = -i$$